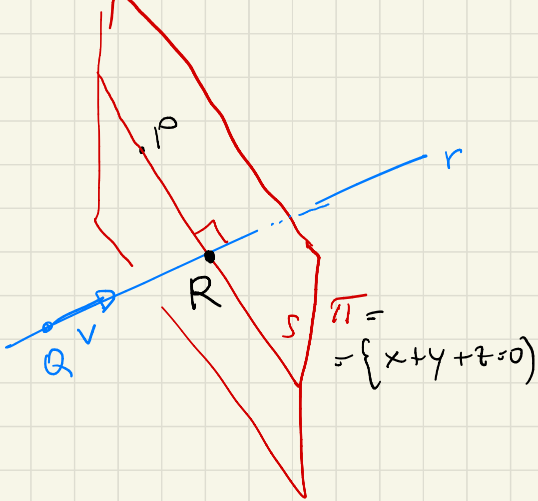


Cerco $\pi \ni P$

$\pi \perp r$

$$r = \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \qquad P = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\pi = \{ x + y + z = d \} \qquad \text{giac}\,\pi = \{ x+y+z=0 \}$$

FASCIO DI PIANI PARALLELI TRA LORO

TUTTI ORTOGONALI A r

Impongo che π contenga P: $-1 + 0 + 1 = d \Rightarrow d = 0$

$$\pi = \{ x+y+z=0 \} \qquad r = \left\{ \begin{pmatrix} 1+t \\ t \\ 2+t \end{pmatrix} \right\}$$

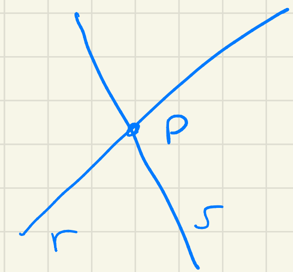
$$1+t+t+2+t=0 \iff 3t+3=0 \iff t=-1$$

$$R=\begin{pmatrix}0\\-1\\1\end{pmatrix} \qquad P=\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

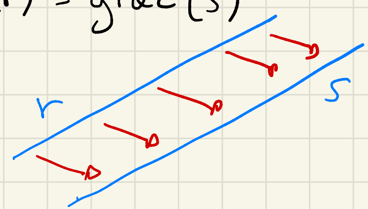
$$s=\left\{P+t\overrightarrow{PR}\right\}=\left\{\begin{pmatrix}-1\\0\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\}$$

Due rette distinte in $\mathbb{R}^3$ possono essere:
r e s

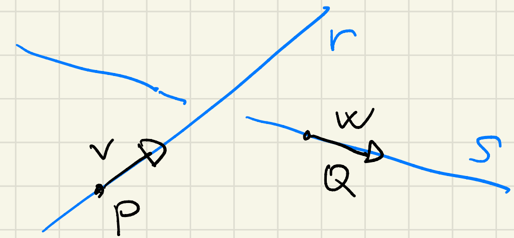
1) <u>incidenti</u> se $r \cap s = \{P\}$



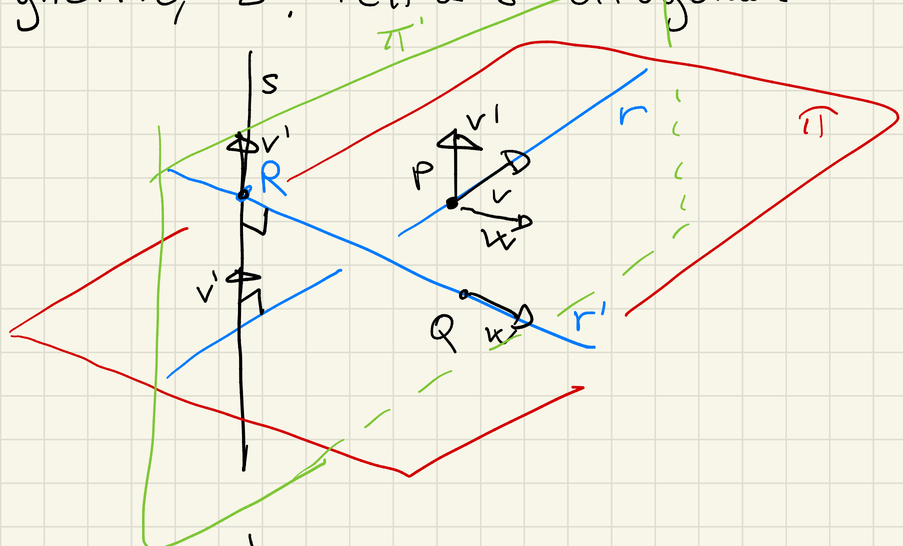
2) <u>parallele</u> se $giac(r) = giac(s)$



3) SGHEMBE se non sono né incidenti né parallele



Prop: Se r e r' sono sghembe, $\exists!$ retta s ortogonale a entrambe



dim:

π = piano parallelo a r' che contiene r

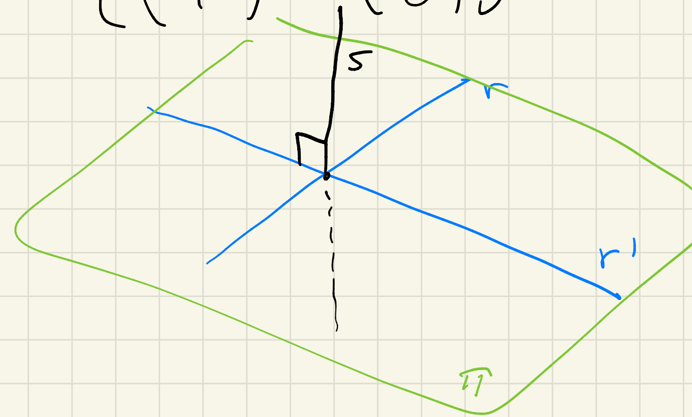
$$\pi = \{P + tv + uw\}$$

v' vettore ortogonale a π $\quad v' = v \wedge w = v \times w$

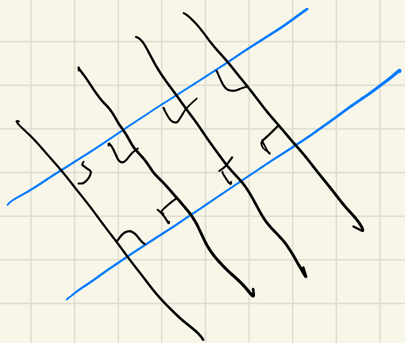
$\pi' = \{P + tv + uv'\}$ $\qquad R = \pi' \cap r' \Rightarrow$ l'unica retta s

ortogonale a r e r' è $\{R + tv'\}$

Esempio: $\quad r = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} \right\} \qquad r' = \left\{ \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + u \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

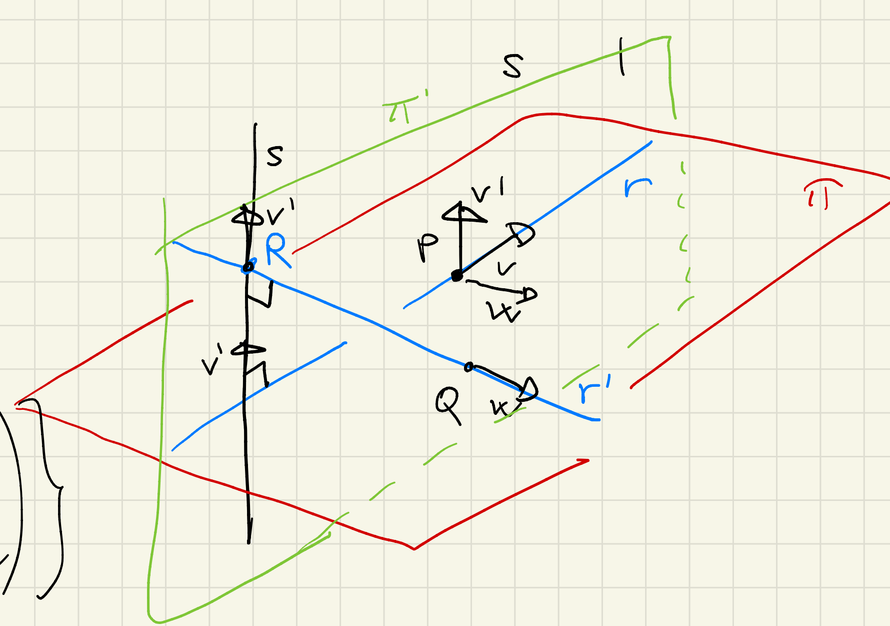


La prop. è vera anche se sono incidenti
(no se sono parallele)



∞ rette
perpendicolari
a entrambe

$$v' = \underset{v}{\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}} \times \underset{w}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$



$$\pi' = \{ P + tv + uv' \}$$

$$= \left\{ \underset{P}{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}} + t \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} + u \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} \right\}$$

$R = \pi' \cap r'$ I modo: $\begin{pmatrix} 1 - 2t - 3u \\ 3u \\ 1 + 3t - 2u \end{pmatrix} = \begin{pmatrix} -2 + s \\ 1 + s \\ 1 \end{pmatrix}$ risolvo s, t, u

II modo: π' in forma cartesiana

$$\pi' = \{ax + by + cz = d\}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} \times \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -9 \\ -13 \\ -6 \end{pmatrix}$$

$$= \{9x + 13y + 6z = d\}$$

$$P \in \pi' : \quad 9 + 6 = d \qquad d = 15$$

$$\pi' = \{9x + 13y + 6z = 15\}$$

$$R = \pi' \cap r'$$

$$9(s-2) + 13(s+1) + 6 = 15$$

$$22s = 15 + 18 - 13 - 6 = 14$$

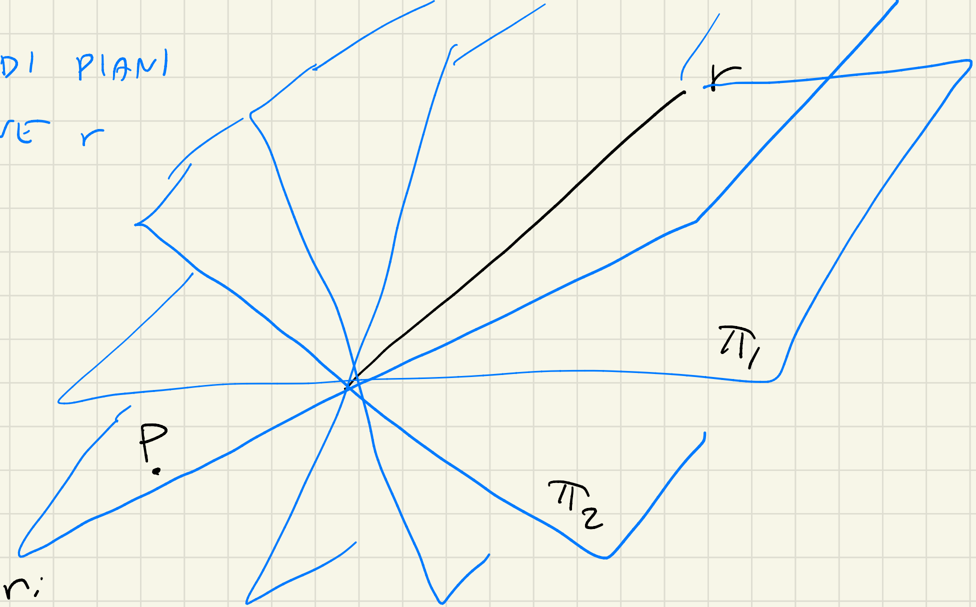
$$s = 7/11$$

$$R = \begin{pmatrix} -15/11 \\ 18/11 \\ 1 \end{pmatrix}$$

$$s = \{R + tv'\} = \left\{ \begin{pmatrix} -15/11 \\ 18/11 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} \right\}$$

FASCIO DI PIANI

CHE CONTIENE r



r in forma <u>CARTESIANA</u>

$$r = \begin{cases} 2x - 3y + z = 1 \\ x + 5y - z = -1 \end{cases}$$

Piano generico che contiene r:

$$r = \begin{cases} 2x - 3y + z - 1 = 0 & \pi_1 \\ x + 5y - z + 1 = 0 & \pi_2 \end{cases}$$

$$t(2x - 3y + z - 1) + u(x + 5y - z + 1) = 0$$

$t = 1,\ u = 0 \longrightarrow 2x - 3y + z - 1 = 0 \quad \pi_1$

$t = 0,\ u = 1 \longrightarrow x + 5y - z + 1 = 0 \quad \pi_2$

Trova π che contiene r e $P = \begin{pmatrix} 7 \\ 0 \\ 5 \end{pmatrix}$. Sostituisco P nell'equazione del fascio

$$t(14 + 5 - 1) + u(7 - 5 + 1) = 0$$

$$18t + 3u = 0$$

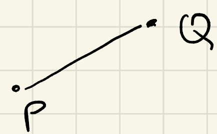
$$6t + u = 0 \qquad t = 1 \quad u = -6$$

$$2x - 3y + z - 1 - 6(x + 5y - z + 1) = 0$$

$$\boxed{-4x - 33y + 7z - 7 = 0}$$

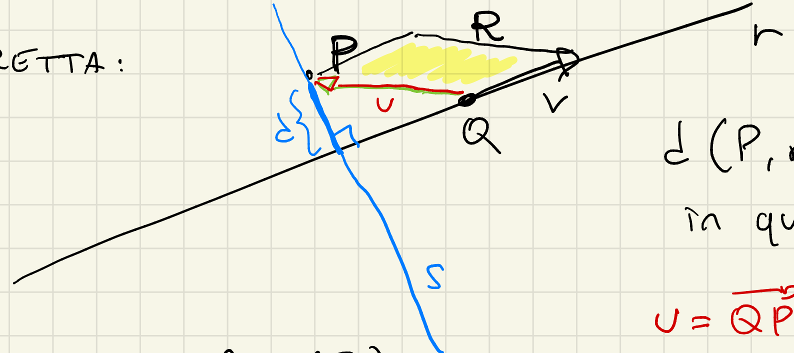
DISTANZE

FRA PUNTI:



$$d(P, Q) = \|\vec{PQ}\|$$

FRA PUNTO E RETTA:



d(P, r) è definita in questo modo

$u = \overrightarrow{QP}$

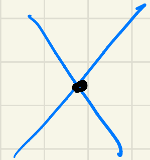
Prop: $d = \dfrac{\| u \times v \|}{\| v \|}$ ← $\dfrac{\text{Area}(R)}{\text{base}(R)} = \text{altezza}(R) = d$

Esempio: $P = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad r = \left\{ \underset{Q}{\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}} + t \underset{V}{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}} \right\}$ $\quad u = \overrightarrow{QP} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

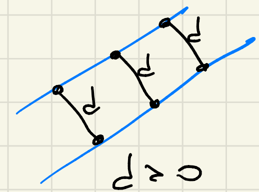
$$d = \frac{\left\| \overset{P}{\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}} \right\|}{\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

DISTANZA FRA DUE RETTE : minima distanza fra punti delle due rette

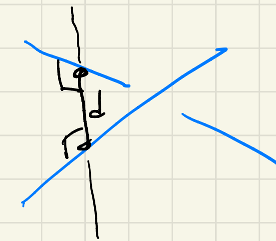
in alternativa:



$d = 0$



$d > 0$



Prop:

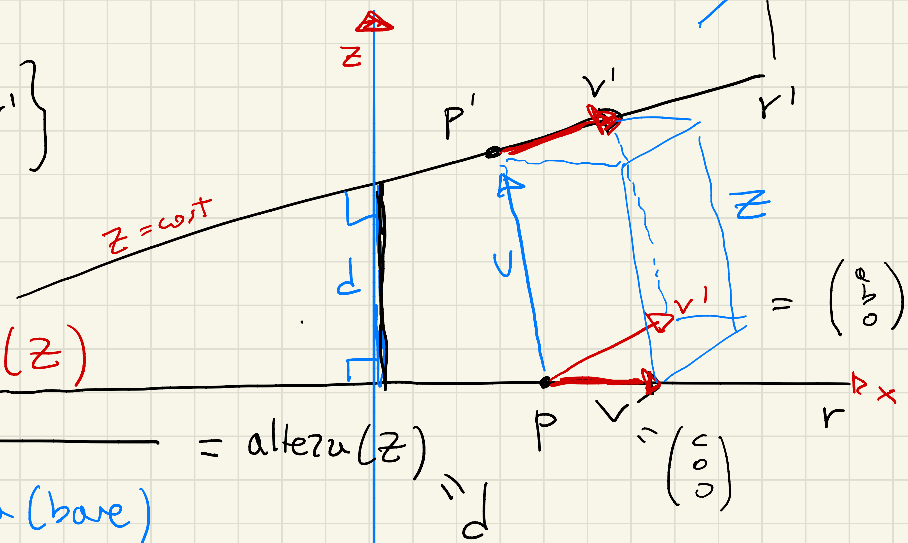
r e r' sghembe:

$r = \{P + tv\}$, $r' = \{P' + tv'\}$

$u = \overrightarrow{PP'}$

$$\boxed{d = \frac{|\det(u\ v\ v')|}{\| v \times v' \|}}$$

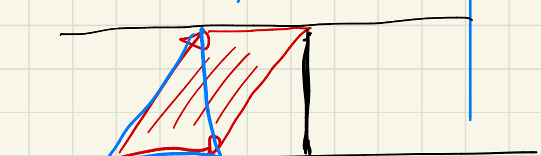
$$= \frac{\text{Vol}(Z)}{\text{Area}(\text{base})} = \text{altezza}(Z) = d$$



(sapendo che non sono parallele)

Cor: r e r' incidenti

$\Leftrightarrow \det(u\ v\ v') = 0$

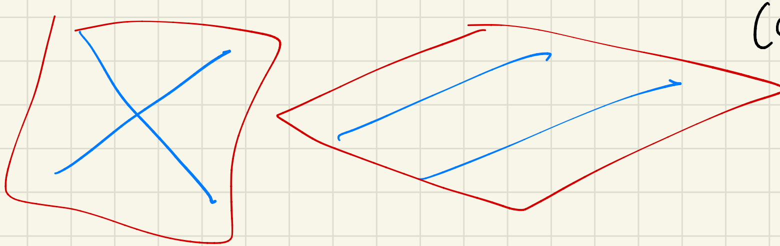


Esercizio: $r = \left\{ \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$, $r' = \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$

Calcolare d.

Con: $r = \{ P + tv \}$, $s = \{ Q + tw \}$

r e s sono incid. o parall. $\iff$ sono complanari $\iff$
(contenute in un piano)



$$\det \begin{pmatrix} v & w & u \end{pmatrix} = 0$$

$u = \overrightarrow{PQ}$

Esercizi:

9.1 - 9.8